Sedimentary Geology 232 (2010) 119–129

Contents lists available at ScienceDirect

Sedimentary Geology

journal homepage: www.elsevier.com/locate/sedgeo

The planar shape of drumlins

Matteo Spagnolo [a,*], Chris D. Clark [b], Anna L.C. Hughes [c], Paul Dunlop [d], Chris R. Stokes [e]

[a] Geography and Environment Department, University of Aberdeen, United Kingdom
[b] Department of Geography, University of Sheffield, United Kingdom
[c] School of the Environment and Society, Swansea University, United Kingdom
[d] School of Environmental Sciences, Ulster University, United Kingdom
[e] Department of Geography, Durham University, United Kingdom

A R T I C L E I N F O

Article history:
Received 9 July 2009
Received in revised form 5 January 2010
Accepted 27 January 2010
Available online 11 April 2010

Keywords:
Glacial geomorphology
Quantitative geomorphology
Drumlin
Planar shape
Lemniscate loop
GIS
Remote sensing

A B S T R A C T

The asymmetry of the planar shape of drumlins is an established paradigm in the literature and characterizes drumlins as resembling tear drops with a blunt (bullet-shaped) stoss end and a tapering (pointed) lee end. It is widely cited and never been seriously questioned. In this paper, the planar shape of 44,500 drumlins mapped in various regional settings from drumlin fields in North America and Northern Europe were objectively analysed by means of Geographic Information System tools. Two parameters were considered. The first (denoted here as As_{pl}) focuses on the relative position of the point of intersection between the axes of the maximum length and the maximum width. It is defined as the distance between the upstream (i.e. beginning of the drumlin) and the intersection point (measured along the longitudinal axis) divided by the entire length of the long axis. Results indicate that the intersection point of the majority of drumlins (64%) is very close to the longitudinal midpoint ($0.33 < As_{pl} < 0.66$). The second parameter (As_{pl_A}) is defined as the ratio between the area of the upstream half of the drumlin to that of the entire drumlin. Results show that for most drumlins (81%), the upper half area is almost as large as the down-half ($0.45 < As_{\mathrm{pl}\_A} < 0.55$). Taken together, these results concordantly indicate that drumlin planar shape has a strong tendency to be longitudinally symmetric and that the long-established paradigm of their plan form is false.

1. Introduction

Drumlins are subglacial landforms that can be described as rounded hills, usually about 600 m long and 200 m wide (Clark et al., 2009). They are found in all landscapes formerly occupied by ice sheets and they typically form fields of hundreds or thousands of associated drumlins. Within a field, drumlins are usually elongated parallel to the inferred palaeo-ice flow direction and two or more ice flow direction can create superimposed landforms (Clark, 1993).

The drumlin is probably the most studied landform in glacial geomorphology, with a literature that totals over 1400 papers on the topic. They have been recognized in many formerly glaciated landscapes and several thousands of these landforms have now been mapped and studied worldwide (e.g. Gluckert, 1973; Zakrzewska Borowiecka and Erickson, 1985; Francek, 1991; Hattestrand et al., 1999; Hess and Briner, 2009). Extensive reviews of the drumlin literature have summarized the main characteristics of drumlins and highlighted some questions that remain unanswered (Menzies, 1979a; Patterson and Hooke, 1995). Among them, the most crucial appears to be drumlin genesis, which, at present, is still not entirely explained. However, several competing hypotheses have been formulated (e.g. Fairchild, 1929; Smalley and Unwin, 1968; Shaw and Freschauf, 1973; Boulton, 1987; Shaw, 1989; Dardis and Hanvey, 1994; Hindmarsh, 1999; Fowler, 2000), most of which seek to explain aspects of drumlin sedimentological and/or morphological characteristics. Because drumlins have been studied for almost two centuries, many characteristics of their morphology were expressed a long time ago and sometimes based on perhaps pre-conceived ideas and assertion made, drawn from relatively small sample of landforms. With the advent of Geographical Information System (GIS) tools and the increasing availability of high resolution Digital Terrain Models (DTM) and satellite images, it is now possible to extensively map formerly glaciated terrains worldwide and to quantitatively assess, on larger sample sizes, many of these widely accepted paradigms concerning drumlin morphology.

One crucial aspect of drumlin morphology is their plan form (i.e. planar shape), known to be elongated according to the ice flow direction and often described as longitudinally asymmetric, with a rounded and wider upstream (stoss) portion and a more tapered downstream (lee) end (Fig. 1). This property has played a significant role in both the development of some drumlin formation theories and the reconstruction of the palaeo-ice flow direction within drumlin fields. Many theories of drumlin formation have been influenced by

* Corresponding author.
E-mail addresses: m.spagnolo@abdn.ac.uk (M. Spagnolo), c.clark@sheffield.ac.uk (C.D. Clark), a.hughes@swansea.ac.uk (A.L.C. Hughes), p.dunlop@ulster.ac.uk (P. Dunlop), c.r.stokes@durham.ac.uk (C.R. Stokes).

0037-0738/$ – see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.sedgeo.2010.01.008

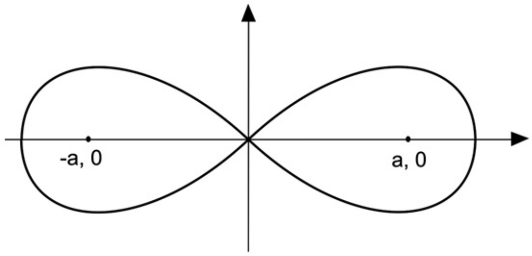

Fig. 1. The classic lemniscate geometric figure as defined by Bernoulli in 1694. Half of the lemniscates loop does correspond to the classically described stoss–lee shape of drumlins, with ice flow from the wider portion to the tapered end. If the intersection between the two loops represent the Cartesian point (0,0), then the two foci (black dots) have coordinates ($-a$, 0) and (a, 0). The curve is then defined by the locus of a point the product of whose distances from the foci is equal to a^2. Drumlins were first compared to half a lemniscate loop by Chorley (1959). Note that the two foci represent the intersections between what are commonly known as the drumlin longitudinal and transverse axes.

this planar shape asymmetry, particularly those that suggest drumlins are formed by sediment accumulation on the lee side of an obstacle of some nature (ice, e.g. Schomacker et al., 2006; frozen till, e.g. Armstrong and Tipper, 1948; stiffer till, e.g. Slater, 1929; rock, e.g. Chamberlin, 1883). From this perspective, the overall drumlin planar shape would be expected to be asymmetric because of the large obstacle on the stoss side (more or less covered by till) and the tapering accumulation of material downstream in its pressure-shadow. In palaeo-glaciological reconstruction, although it is not always expressly stated, the asymmetric shape (both bi- and three-dimensional) of drumlins, often in conjunction with other subglacial landforms morphometry, has been likely used to infer the ice flow direction (e.g. Dyke and Prest, 1987; Kleman et al., 1997; Clark and Meehan, 2001).

Although often viewed as one of the key defining features of drumlins and despite its importance for genetic implications and for palaeo-glaciological reconstruction, there has never (to our knowledge) been any attempt to quantitatively analyse the asymmetry of drumlin planar shape, at least over a large sample of landforms. In this paper we present the results of statistical analysis of drumlin planar shape from a database of 44,500 drumlins mapped in Northern America and Northern Europe in order to answer the following questions:

- What is the most common planar shape of drumlins?
- What proportion of drumlins in a given field shows the classic asymmetric shape?
- Does asymmetry match with the palaeo-ice flow direction?

2. Review of drumlin planar shape

In the early literature, drumlin planar shape has been variously described as lenticular (Hitchcock, 1876), elliptical (Chamberlin, 1883), elliptical to sub-circular (Kupsch, 1955; Barnett and Finke, 1971; Wright, 1962; Reed et al., 1962) and oval (Charlesworth, 1957). Comparisons to bodies of more or less regular geometries are various: half torpedo (Alden, 1905), half egg, inverted bowl of spoon (Flint, 1957), baguette (Rouk and Raukas, 1989), tear drop, cigar (Ebers, 1926; Ebers, 1937) or spindle (Shaw, 1983). However, the most significant paper on drumlin planar shape is probably that of Chorley (1959). In this paper, drumlin shape is compared to that of an airfoil and described in terms of the geometric figure known as the lemniscate ("pendant ribbon" in Latin) loop. This geometric figure was defined by Bernoulli in 1694 as the locus of a point, the product of whose distances from the foci (($-a$, 0) and (a, 0)), which are 2a units apart, is equal to a^2, (Fig. 1). Chorley had already been successful–a few years earlier–in comparing the shape of fluvial catchments to that of a lemniscate loop (Chorley et al., 1957). Therefore, the shape of drumlins, far more regular than that of fluvial catchments, appeared to more closely resemble the shape of the lemniscate loop.

By considering half of a lemniscate loop, which would represent the drumlin shape, the figure can also be described in simple polar-coordinate form by the equation: $\rho = l \cos k\theta$, where ρ is the radius, θ is the polar angle, l is the length of the drumlin and $k = (l^2\pi)/4A$, where A is the area of the drumlin. After asserting that the lemniscate is an ideal proxy for drumlin planar shape, Chorley (1959) specifically focussed on the k parameter and its interpretation. When drumlins have a large k value (long and slender drumlins), their shape was thought to offer little resistance to the ice flow, which means that they are more likely to be compatible with fast flowing ice.

Although not based on any quantitative data, Chorley's comparison of drumlin shape to the lemniscate loop became very popular, probably for these two main reasons. First, his paper was published at the beginning of the quantitative revolution in geomorphology. In that context, the idea of comparing drumlin shape to a perfect geometric figure described by a simple equation was extremely appealing. Secondly, in his paper, Chorley emphasized the importance of the k parameter in the lemniscate loop equation. This measurable parameter appeared to have crucial implications for the interpretation of palaeo-ice dynamics, especially ice velocity. With some exceptions (Doornkamp and King, 1971; Trenhaile, 1975), most authors who later studied k did not realize that this parameter represents nothing else but a proxy for the elongation of a drumlin. This is because k is the ratio between the length and the area of a drumlin and since the area is generally a function of the length times the width, k is essentially a function of the length divided by the width, which is the elongation. Chorley was, of course, correct and quite ahead of his time when he suggested interpreting k (basically drumlin elongation) in terms of ice velocity (see later confirmation of this in papers by Hart, 1999 or Stokes and Clark, 2002). However, an unfortunate outcome of this work (as we will see later) was the unjustifiable forcing of drumlin planar shape into that of a lemniscate loop, thus instigating the paradigm of an asymmetric longitudinal shape of drumlins; the now classical stoss–lee form.

Following Chorley's (1959) paper, many authors evaluated the parameter k of the lemniscate equation for various drumlin fields worldwide, with mean values ranging from 2.5 to 5 (Barnett and Finke, 1971; Smalley and Unwin, 1968; Gravenor, 1974; Trenhaile, 1975; Zakrzewska Borowiecka and Erickson, 1985; Piotrowski, 1987; Harry and Trenhaile, 1987; Mitchell, 1994; Wysota, 1994; Rattas and Piotrowski, 2003). Meanwhile, the idea that drumlin planar shape shows a consistent longitudinal asymmetry, already expressed by others (e.g. Armstrong and Tipper, 1948) and strongly supported by the comparison with the lemniscate loop, became widely accepted. In text books and reviews published after the 1950's drumlins are described as having their transverse axes upstream from the midpoint of the long axes (Flint, 1971) or as being characterized by a wider up-ice end and a pointed down-ice end (e.g. Menzies, 1979b; Benn and Evans, 1998, p. 431).

The only exceptions and criticisms to the paradigm of an asymmetric drumlin planar shape are those by:

1. Barnett and Finke (1971)); who found considerable departure from the classic airfoil shape described by Chorley.
2. Reed et al. (1962); who suggested that drumlin 3D form should be compared with that of (half of) an ellipsoid, thus with an elliptical planar shape.
3. Shaw et al., who repeatedly described, among other forms, drumlins characterized by a shape opposite to the classic one, with a pointy stoss and a broad lee end (Shaw, 1983; Shaw and Kvill, 1984; Shaw et al. 1989; Shaw, 2002).
4. Smalley and Warburton (1994, p. 243); who highlighted how some authors might have favoured the lemniscate over the ellipsoid shape mostly because of the "the elegance of its polar-coordinate equation".

Overall, it is remarkable to notice how not one of the mentioned papers (for or against the lemniscate loop idea) has ever attempted a proper quantitative analysis of the actual planar shape of drumlins.

3. Methods

Drumlins were mapped using their bounding break of slope (as GIS vector polygons) from Digital Terrain Models (DTMs) and satellite images (Table 1). Relief shading is the most common technique for enhancing landforms on DTMs (e.g. Clark and Meehan, 2001), however users need to be aware of the problems caused by azimuth-biasing (Smith et al., 2001; Smith and Clark, 2005) when interpreting landforms. For this reason, although different illuminated renditions of the DTMs were generated to aid identification of the landforms, the final mapping was always done using non-azimuth biased images. Landsat 7 TM winter scenes were used as the satellite source for our mapping. Different satellite images, created either from the combination of different bands or from the panchromatic band, were analysed or interpreted (see Clark et al., 2009 for further methodological details). Although drumlins were mapped by different people in various regions worldwide, mapping was always cross-checked by at least one of the other investigator. More than 33,000 drumlins were mapped in Britain (several 10's of individual drumlin fields) and Ireland (the Bann Valley drumlin field in Northern Ireland) on 5 m DTMs (Fig. 2), while more than 15,000 drumlins were mapped in the drumlin fields of Alta (Norway), Tana (Finland), Ungava (SE Canada), and Keewatin (central Canada) on Landsat7 TM images (15–30 m resolution) (Fig. 3). In the Keewatin region two distinct drumlin fields were mapped, one comprising relatively long and narrow drumlins interpreted as the product of a faster ice flow (Keewatin F) and the other more typical drumlins suggesting a slower ice flow (Keewatin S) (Stokes and Clark, 2003). The database was then automatically filtered in order to specifically avoid cross cutting drumlins, whose perimeter, and therefore shape, is often uncertain and would have required a time-consuming manual analysis. This left 44,500 drumlins whose shape was analysed by means of various GIS techniques.

For each drumlin field the palaeo-ice flow direction was inferred from elements other than the drumlin planar shape (e.g. esker tributaries, configuration of moraine belts, drift dispersal, etc.). In most cases the direction appeared obvious in the context of the known regional ice sheet reconstruction and proximity to the margin.

Drumlin planar asymmetry and shape were quantified by means of two parameters specifically designed for this purpose. The first parameter, denoted as "As_{pl}", was inspired by Flint's (1971) description of an ideal drumlin as having its (longest) transverse axis situated upstream from the midpoint of the (longest) longitudinal axis. Drumlin longitudinal axis (L) was automatically derived with a GIS tool as the longest straight line within the mapped drumlin polygon, see Fig. 4(application available at http://www.jennessent.com/arcview/longest_lines.htm). A second tool was applied to identify the transverse axis (T), derived as the longest perpendicular straight line to the longitudinal axis within the drumlin polygon. This second, *ad-hoc* developed, tool creates a (very large) number of evenly distributed points along L and for each point derives a line perpendicular to L and delimited by the drumlin polygon outline. The length of each of these lines is then measured and only the longest line is saved as T. This is, of course, only one of several possible ways of deriving the longitudinal and the transverse axes of a drumlin. This specific technique was chosen here because (i) it represents a relatively easy automated procedure, (ii) gives results that, when tested, always appeared very similar to what one would manually draw for L and T, and (iii) is also very effective for the size of the database analysed. Possible errors could derive from mapped landforms that represent only a portion of the original drumlin (eroded drumlins) or from drumlins with very irregular shapes. The first case is relatively rare, and could only account for a small percentage of the mapped drumlins because such landforms were avoided during mapping. Irregular shapes, although still relatively rare, can sometimes be found within a swarm of drumlins, mostly resulting from the superimposition of a new set of landforms over an old one. It is for this reason that particular care was taken, when mapping, in identifying all recognizable compound (superimposed/crosscutting) drumlins. These complex landforms were not included in the present analysis.

For each drumlin field, the reconstructed (not based on drumlin shape) direction of palaeo-ice flow made it easy to identify the upstream and downstream end points along the longitudinal axis of each drumlin (A and C in Fig. 4). The point of intersection (B in Fig. 4) between the longitudinal and transverse axes was also recorded. With these three points in mind (A, B and C in Fig. 4), the asymmetry of drumlin planar shape was evaluated as $As_{pl} = AB/AC$. Given that As_{pl} varies between 0 and 1, it would generally be expected that when $As_{pl} = 0.5$ drumlins should be close to symmetrical; when approaching zero drumlins should have the classic asymmetry described in the literature (wide stoss end up-ice and a tapered lee end facing down-ice); and when As_{pl} is near to 1 drumlins should show an opposite asymmetry (which for convenience we term "reversed drumlins") (Fig. 4).

This technique really only considers the relative position of the intersection between the longitudinal and transverse axes, which might not always be an ideal proxy for the grade of asymmetry that characterizes the planar shape of a drumlin. In fact, the same As_{pl} value of 0.5 could correspond to very different drumlin planar shapes, as shown in Fig. 5.

A better approach, therefore, is to divide the planar surface of the drumlin into two halves by a line perpendicular to the longitudinal axis and intersecting its midpoint. Drumlin planar asymmetry can now be quantified by the parameter As_{pl_A}, which corresponds to the ratio between the upstream half area and the total area of the drumlin ($As_{pl\_A} = A_{up}/A_{tot}$) (Fig. 5). Without taking into account unrealistic shapes, As_{pl_A} is expected to range between approximately 0.3 and 0.7, with high values indicating upstream halves larger than downstream halves (classically asymmetric drumlins), values close to 0.5 indicating upstream halves as large as downstream halves (symmetric drumlins) and low values indicating upstream halves smaller than downstream halves (reversely asymmetric drumlins) (Fig. 5).

4. Results

4.1. Parameter As_{pl}

Fig. 6 plots the As_{pl} frequency distribution of all drumlins ($n = 44{,}500$) revealing a normal distribution with a mean value of 0.47, a standard

Table 1
Metadata of the analysed drumlins. From first to last column, the name of the studied region, the number of drumlins analysed, the imagery used for mapping, its spatial resolution and source.

Region	N	Imagery used	Spatial resolution	Source
Britain (United Kingdom)	30,000	DTM	5 m	NEXTMAP Britain, InterMap Technologies© BGS(NERC)
Alta (Norway)	1700	Satellite image	15/30 m	Landsat 7 TM
Tana (Finland)	800	Satellite image	15/30 m	Landsat 7 TM
Bann Valley (United Kingdom)	3500	DTM	5 m	Ordinance Survey Northern Ireland (OSNI) enhanced DTM
Ungava (Canada)	6000	Satellite image	15/30 m	Landsat 7 TM
Keewatin F (Canada)	1000	Satellite image	15/30 m	Landsat 7 TM
Keewatin S (Canada)	1500	Satellite image	15/30 m	Landsat 7 TM

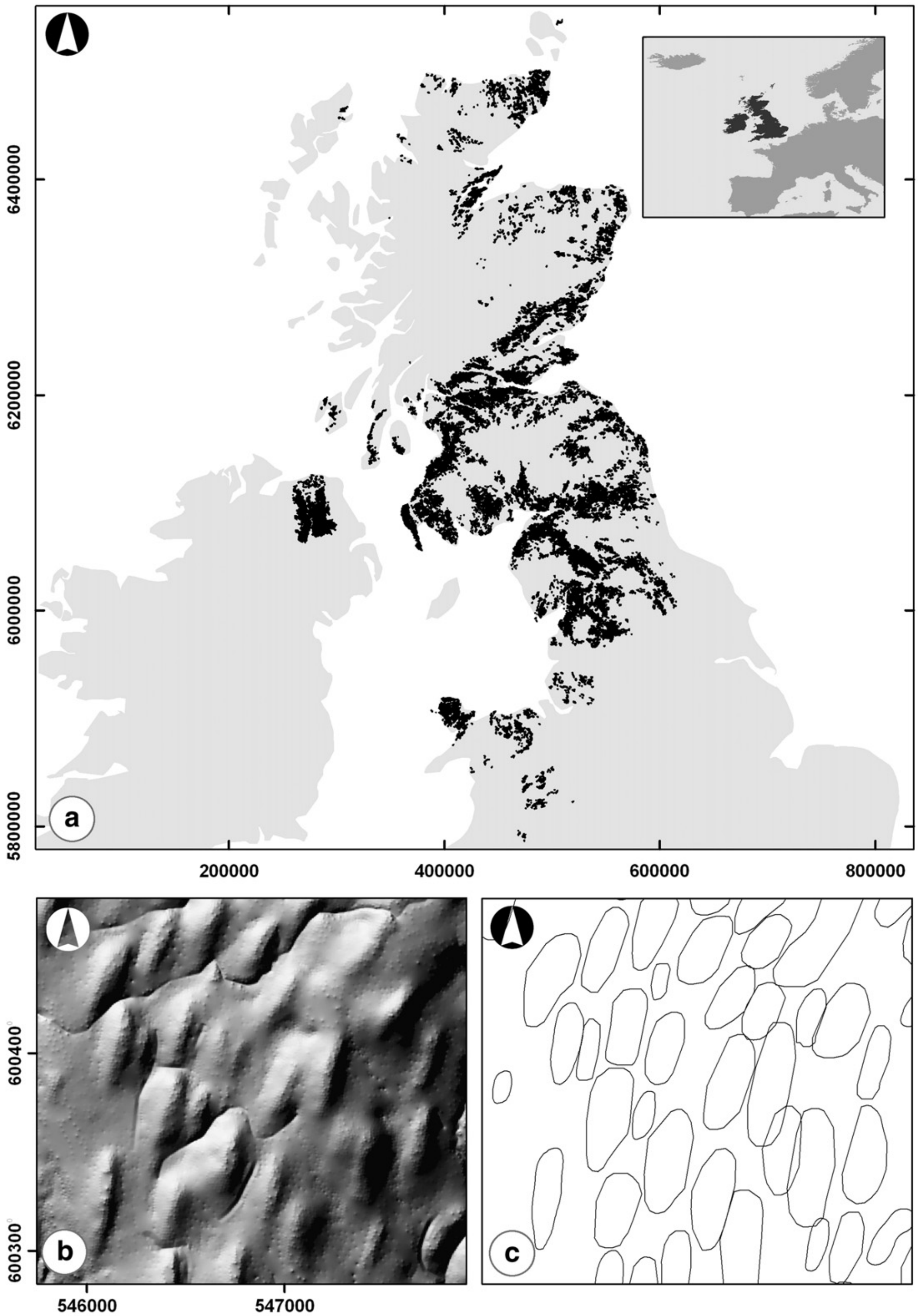

Fig. 2. A map (a) showing drumlins mapped in the UK (all drumlin fields of Britain and the Bann Valley field in North Ireland) from 5 m DTMs. The lower images are an example of mapping from the Yorkshire Dales area: a shaded image of the DTM (b) and the corresponding mapped drumlins (c). Note that the actual mapping of (c) results from the interpretation of at least three differently shaded images, one of which is that shown in (b). Coordinate system is in UTM (zone 30N), Datum is WGS84.

deviation of 0.17, and approximately equal tails and with little skewness (0.08). If the same analysis is repeated within each mapped region, the average As_{pl} values are all very similar and always relatively close to 0.5 (Table 2).

The entire range of As_{pl} values can be subdivided into 3 main intervals: lower than 0.33, an interval indicating drumlins whose transverse axis intersects the longitudinal axis close to the longitudinal axis startpoint (here called “classic drumlins” for simplicity), 0.33–0.66 (“symmetric drumlins”) and higher than 0.66 (“reversed drumlins”). The frequency distribution within these three intervals show that 64% of all mapped drumlins have an As_{pl} value between 0.33 and 0.66, typical of symmetric drumlins while only 22% have an As_{pl} value lower than 0.33 (classic) and 14% higher than 0.66 (“reversed) (Fig. 7). Similar distributions are apparent within each mapped region, with the

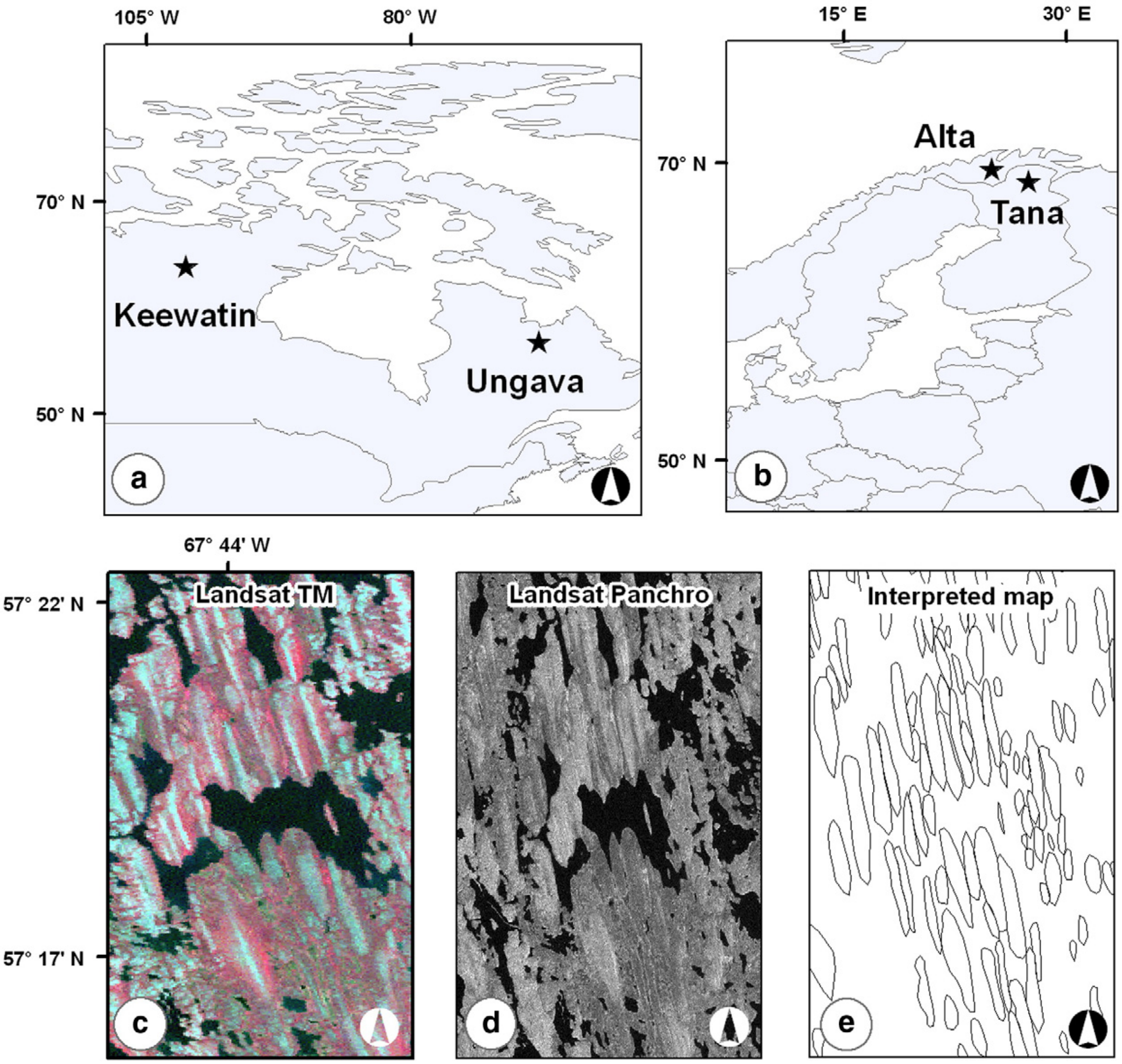

Fig. 3. The location of drumlin fields (black stars in the upper images) mapped from satellite images and an example of mapping from the Ungava drumlin field in Canada (lower images). The upper left image (a) shows the approximate locations of the two Keewatin drumlin fields and the Ungava drumlin field. The upper right image (b) shows the location of the Alta drumlin field in Norway and the Tana drumlin field in Finland. The three lower images, from left to right, are a Landsat7 ETM + image (gray-shaded: R, G, B, 4, 3, 2) of a small portion of the Ungava drumlin field (c), the equivalent Landsat panchromatic image (d) and the actual mapping of the same area (e). Coordinate system is in latitude/longitude degrees, Datum is WGS84.

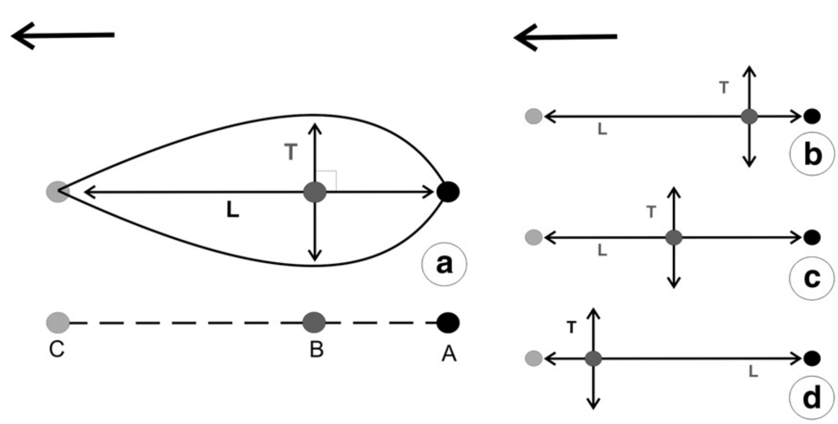

Fig. 4. The sketch on the left (a) shows how As_{pl} is calculated: first the longest line within a drumlin is drawn (longitudinal axis, *L*), then the longest line perpendicular to *L* is drawn (transverse axis, *T*) and the point B, intersection of *L* and *T*, is recorded. Given the ice flow direction (arrow) the upflow point A and the downflow point C along *L* are identified. As_{pl} is then calculated as AB/AC. The sketch on the right shows three possible cases: the top one (b) would be a classic drumlin, with an $As_{pl} = 0.2$. The middle one (c) represents a perfectly symmetric drumlin, with an $As_{pl} = 0.5$. The bottom one (d) is a reversed drumlin, with an $As_{pl} = 0.8$.

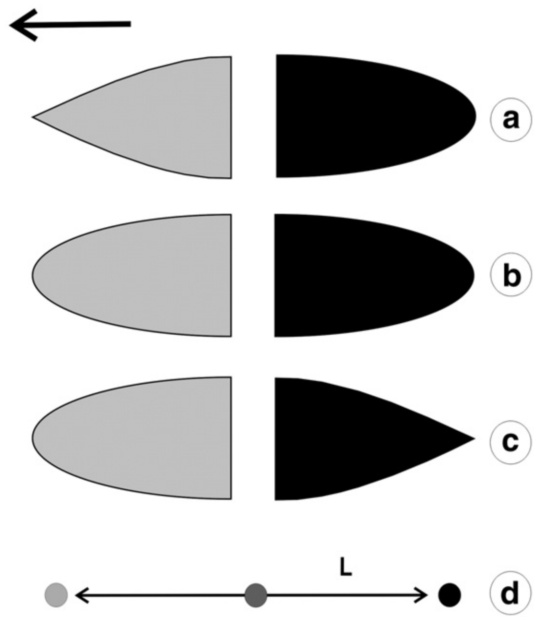

Fig. 5. Three sketches representing drumlins with very different planar shapes: (from the top) a classical asymmetric drumlin (a), a symmetric drumlin (b) and a "reversed drumlin" (c). However, they all share the exact same value of As_{pl} (= 0.5), since for all of them the intersection between L and T falls half way through L (d). To resolve this issue, it is possible to measure drumlin planar shape in terms of surface areas. Drumlins are split into two halves according to the midpoint along their longitudinal axis (see lowermost sketch). As_{pl_A} is then calculated as the ratio between the upflow half area (in black in the above sketches) and the total drumlin area (black + gray). In this case, the top (classic) drumlin has an As_{pl_A} value of 0.65, the mid (symmetric) drumlin an $As_{\mathrm{pl}\_A} = 0.5$ and the lower (reversed) drumlin an $As_{\mathrm{pl}\_A} = 0.35$.

exception of the Keewatin F field where classic and symmetric drumlins are present in similar percentages (Fig. 7). It is no surprise, therefore, that this is the region with the lowest average As_{pl} (0.43) (Table 2). At the sub-drumlin field scale, when randomly zooming in on any drumlin field, it appears that most drumlins are symmetrical, while classic and reversed drumlins are both present, but in much lower percentages, see Fig. 8. Moreover, the spatial distribution of As_{pl} does not show any specific pattern, and different planar shapes can be found next to each other (Fig. 8).

4.2. Parameter As_{pl_A}

For our second measure of asymmetry, based on the relative areas of drumlins halves, we find a similar, normally-distributed, frequency distribution (Fig. 9) with a mean of 0.51 (and standard deviation of 0.04). Similar As_{pl_A} mean values are obtained for each mapped region and within each drumlin field (Table 2). For simplicity, the entire range of As_{pl_A} can be subdivided into three intervals: values of As_{pl_A} lower than 0.45, indicating drumlins with an up-half area sensibly smaller than the down-half area (here called "reversed drumlins"); values between 0.45 and 0.55, where the area of the up-half drumlin is almost equivalent to the area of the down-half drumlin ("symmetric drumlins"); and values higher than 0.55, characteristics of those drumlins whose up-half areas are substantially larger than their down-half one ("classic drumlins"). The frequency distribution within these three intervals shows that 81% of the mapped drumlins have values between 0.45 and 0.55, indicating that most drumlins are indeed symmetrical (Fig. 10). Only 6.5% of drumlins have values lower than 0.45 ("reversed") and 12.5% have As_{pl_A} values higher than 0.55 ("classic") (Fig. 10). Similar distributions of these three classes are apparent in the different mapped regions, with the exception of the Tana (FIN) and Alta (NOR) drumlin fields, where classic drumlins have a frequency closer, but still lower, to that of the symmetric drumlins (Fig. 10).

In a similar manner to the parameter As_{pl}, As_{pl_A} does not show any particular spatial trend within a drumlin field and different types of drumlins can in fact be found next to each other, see Fig. 11.

Table 2
Average values of the two analysed parameters from the different study areas. First column indicates the geographic regions; N is the number of mapped drumlins; As_{pl}, As_{pl_A} as defined in the text. In all cases except Britain, each region corresponds to a single drumlin field. For simplicity, all British drumlin fields are here treated together as one record, but they all separately show similar average values.

Region	N	Mean As_{pl}	Mean As_{pl_A}
Britain (United Kingdom)	30,000	0.47	0.51
Alta (Norway)	1700	0.44	0.54
Tana (Finland)	800	0.44	0.54
Bann Valley (United Kingdom)	3500	0.48	0.51
Ungava (Canada)	6000	0.5	0.49
Keewatin F (Canada)	1000	0.44	0.52
Keewatin S (Canada)	1500	0.43	0.51
All drumlin fields together	44,500	0.47	0.51

5. Discussion

5.1. Drumlin planar shape

According to the wider literature (reviewed in Menzies, 1979a) and text books (e.g. Benn and Evans, 1998, p. 431), drumlin planar shape is longitudinally asymmetric, with a larger and rounded upstream end and a tapered downstream end. Drumlins should therefore have their

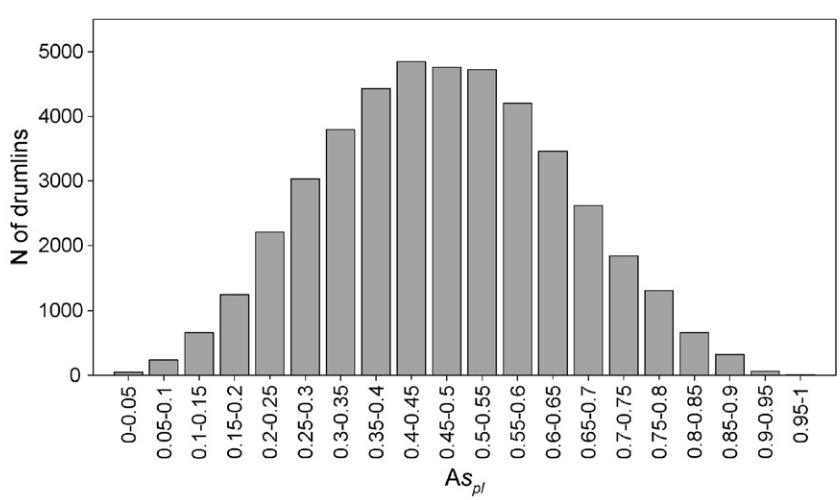

Fig. 6. Frequency (number of drumlins) histogram of drumlin parameter Aspl in classes of 0.05 As_{pl}-units. Note the almost perfect Gaussian distribution.

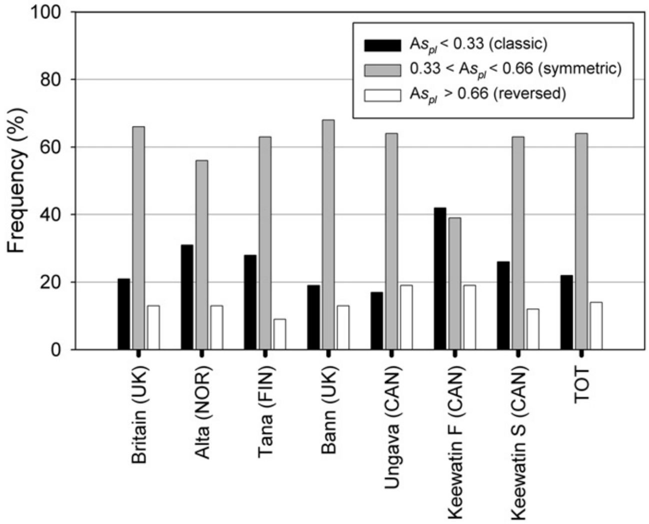

Fig. 7. The frequency (%) of the three main classes of As_{pl} in the different mapped regions and for the entire database of mapped drumlins ("TOT"). With the exception of the Keewatin F, all others show a strong predominance of the 0.33–0.66 As_{pl} class (symmetric drumlins). Both classic (As_{pl}<0.33) and reversed (As_{pl}>0.66) drumlins are present, but classic ones are normally more frequent (with the exception of the Ungava field). Note that in all cases but Britain, regions correspond to single drumlin fields. For simplicity, all British drumlin fields (10's) are here treated together as one record but they all separately show a similar distribution.

(longest) transverse axis intersecting their (longest) longitudinal axis upstream from the longitudinal axis midpoint. Moreover, it would be expected that drumlin up-half area should be greater than drumlin down-half area.

The results presented in this paper indicate that the longitudinal and transverse axes of most drumlins intersect close to the midpoint of the longitudinal axis (mean As_{pl} = 0.47, compare with 0.5). Moreover, the area of the upstream half drumlin is generally very similar to the area of the downstream half (mean As_{pl_A} = 0.51). These results unequivocally indicate that the majority of drumlins are, indeed, characterized by a symmetrical planar shape and there is no evidence of any recurrent longitudinal asymmetry. This is not only surprising in the light of the mainstream view on drumlin planar shape, but also clearly indicates that traditional comparisons to tear drops or lemniscate loops are incorrect. A minority of drumlins (22%) have their axes intersecting each other closer to the upstream end than to the midpoint of the longitudinal axis. However, although less common (14%), the reverse case is also possible and classically asymmetric drumlins are often found close to reversely asymmetric ones. Similarly, a minority (12.5%) of drumlins have an upstream half area substantially higher (As_{pl_A}>0.55) than the downstream one (classically asymmetric planar shape). However, a smaller percentage (6.5%) of drumlins present the opposite case (reversely asymmetric planar shape) and the two types can be found next to each other within a drumlin field. A major implication of this is that the areal asymmetry of drumlin planar shape cannot be considered as a reliable indicator of palaeo-ice flow direction, although we acknowledge this is rarely used in isolation.

Since drumlins are elongated landforms and have been shown to have a longitudinally symmetric planar shape, one might wonder which geometric figure they resemble the most. Is it a rhombus or an ellipse or a rectangle? In order to answer to this question we also evaluated for each drumlin the ratio between its area and that of the rectangle bounding that drumlin (a parameter here called *SF*, as for "Space Filling"). *SF* values close to 0.5 would indicate a shape similar to that of a rhombus, values near 0.78 that of an ellipse and *SF* values close to 1 would indicate a shape similar to a rectangle. The majority (65.5%) of mapped drumlins are characterized by *SF* values between 0.75 and 0.85. Thus, the shape should be close to that of an ellipse, whilst 25.5% of all drumlins have a *SF* lower than 0.75 and 9% have a *SF* value higher than 0.85 (Fig. 12). The mean *SF* value for all drumlins is 0.78, the exact *SF*

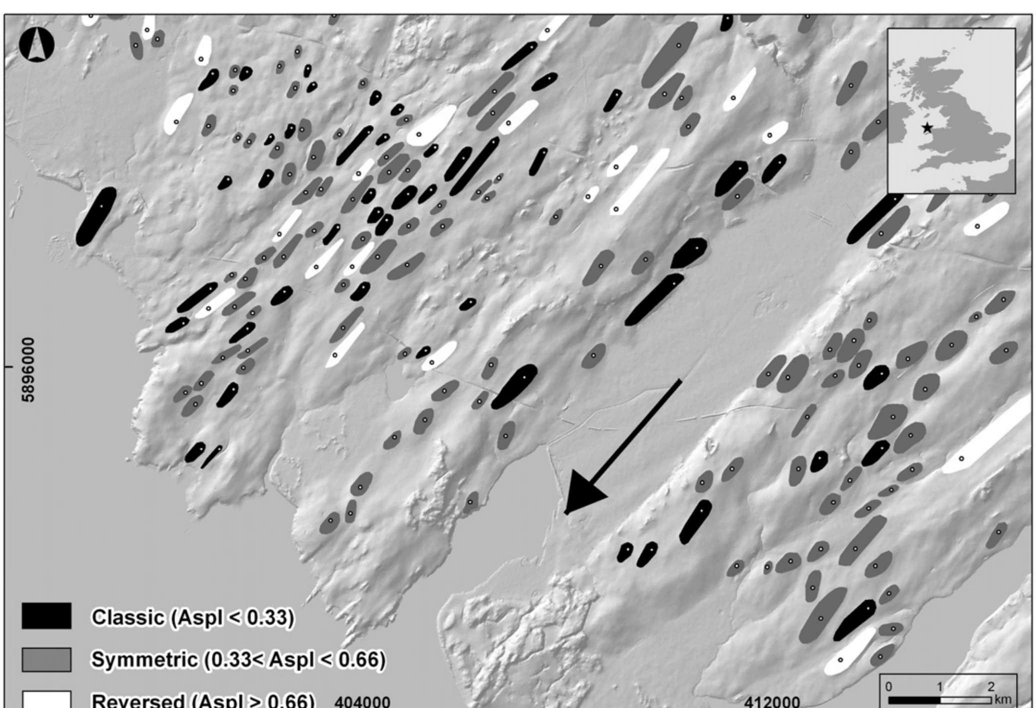

Fig. 8. The spatial distribution of As_{pl} in an inset of the Isle of Anglesey (Britain) drumlin field (a shaded image of the 5 m DTM is used as a background) that can be recognized on the shaded image. The arrow shows the palaeo-ice flow direction. The dot within each drumlin represents the intersection point between the longitudinal and transverse axes. Note that most drumlins are symmetrical but both reverse and classic drumlins are also present. The spatial distribution of the various As_{pl} classes appears random and different drumlin types are found next to each other. Coordinate system is in UTM (zone 30N), Datum is WGS84. Note that only nonoverlapping drumlins are shown in this figure and other non-mapped drumlin-looking landforms were not included in this dataset because made up of bedrock.

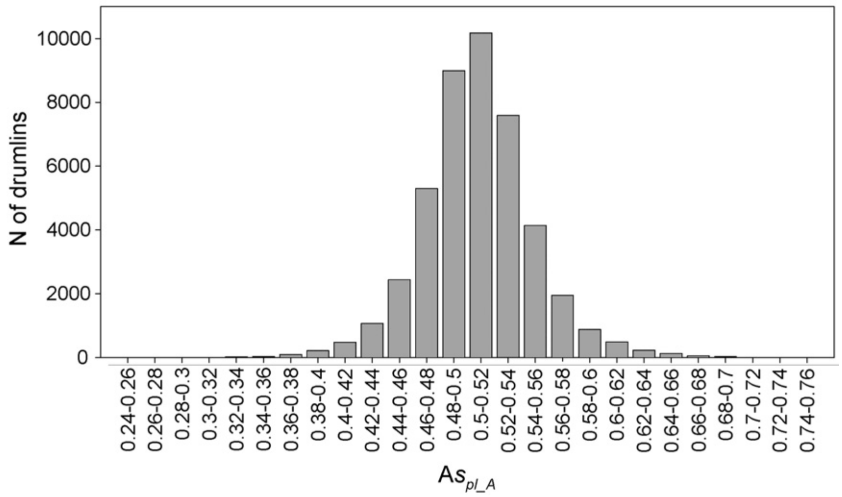

Fig. 9. Frequency (number of drumlins) histogram of drumlin parameter As_{pl_A} in classes of 0.02 As_{pl_A}-units. Note the almost perfect Gaussian distribution.

value of an ellipse. However, it must be acknowledged that departures from the mean values do occur and even when *SF* is exactly that of an ellipse, it is possible that the drumlin outline is still wiggly or irregular.

5.2. Regional scale analysis

Although in most cases the results at the regional or drumlin field scale are very consistent with those obtained by considering the entire database as a whole, some exceptions are present and will be briefly highlighted. It must be stressed, however, that none of these exceptions show a radically different result than the one pictured above, e.g. no one single case shows a vast majority of classically asymmetric drumlins.

The Keewatin F (CAN) drumlin field presents the lowest average value of As_{pl} (although still fairly close to symmetry; −0.43) and it is the only case where classically asymmetric drumlins (As_{pl}<0.33) are (slightly) more frequent than symmetric drumlins. In relative terms, our results indicate that in this specific drumlin field there are more drumlins with their axes meeting less close to the midpoint of the longitudinal axis than in the other analysed regions. It is also interesting to note that this is the only region where an apparent inconsistency between As_{pl} and As_{pl_A} is present. In fact, when the asymmetry of Keewatin F drumlins is measured in terms of areas, the majority of these landforms do show a symmetric shape (the upstream and downstream half areas are generally very similar). The only obvious peculiarity of the Keewatin F drumlin field is that it contains highly elongated landforms that are above the common drumlin elongation average. In fact, this area is within a hypothesised ice stream bed (Stokes and Clark, 2003). However, it is not clear how this could have affected As_{pl} and, in any cases, even when these very elongated drumlins are excluded from the analysis the result does not change significantly.

The Alta (NOR) and Tana (FIN) drumlin fields are both characterized by the highest (although still very close to 0.5) average values of As_{pl_A} (0.54) and the frequency of classic asymmetric drumlins (As_{pl_A}>0.55), although still lower than that of symmetric drumlins, is relatively higher than in other regions. In other words, they fit with the general finding that drumlins are mostly symmetric, but this tendency is less so than found in other analysed regions. Both the Alta and Tana fields were mapped by the same person, so a possible explanation of this somewhat different result could be related to the intrinsic level of subjectivity of the mapping. However, it is more likely that different local factors, yet to be investigated (e.g. geology), could be responsible for the formation of slightly less symmetric drumlins, as such factors have been shown to influence other morphometric aspects of drumlins (Greenwood and Clark, 2010-this issue). One possible explanation is that this Scandinavian region is characterized by relatively hard crystalline bedrock which might have led to the development of more crag-and-tail-like drumlins.

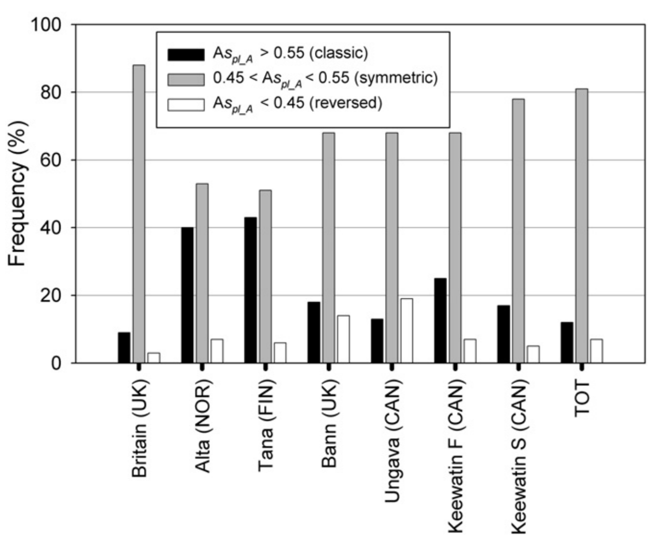

Fig. 10. The frequency (%) of the three main classes of As_{pl_A} in the different mapped regions and for the entire database of mapped drumlins ("TOT"). With the exception of the Tana and Alta drumlin fields, all regions show a strong predominance of the 0.45–0.55 As_{pl_A} class (symmetric drumlins). Both classic (As_{pl_A}>0.55) and reversed (As_{pl_A}<0.45) drumlins are present, but classic ones are normally more frequent (with the exception of the Ungava field).

5.3. Possible biases

At least five possible biases could have affected the study presented here, but they would generally influence the results only at a local scale and most of them can be easily ruled out:

1. The source (in terms of quality, resolution, etc.) of the original data used for mapping may have determined a more or less approximate (and possibly idealized) drawing of drumlin outlines. However, two completely different sets of data were used; high resolution DTMs and lower resolution satellite images, and drumlins mapped from either source give nearly identical results.
2. The planar shape of drumlins could be influenced by some local factors (geology, glacial history, etc.). However, many drumlin

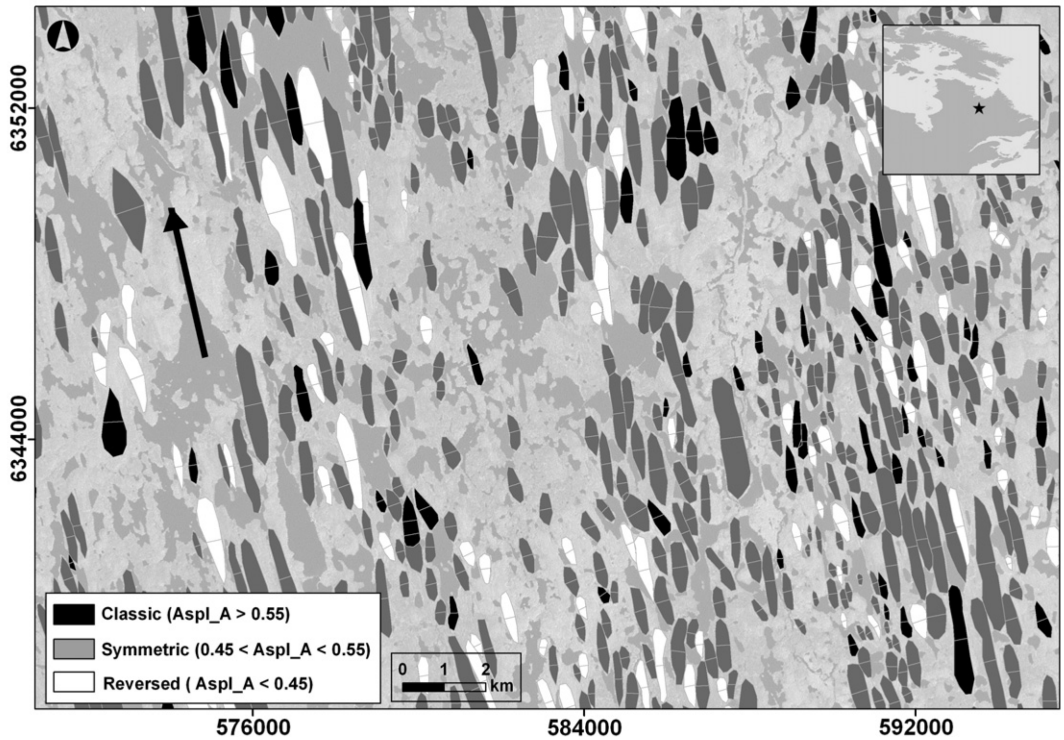

Fig. 11. The spatial distribution of As_{pl_A} in a small region of the Ungava drumlin field (Canada), with the 15 m panchromatic Landsat7 ETM + as a background. For location, see Fig. 3. The arrow (top left) shows the palaeo-ice flow direction. The line across each drumlin shows the position (midpoint of the longitudinal axis) were drumlins were split into two halves for the As_{pl_A} analysis. Note that most drumlins are symmetrical but both reverse and classic drumlins are also present. The spatial distribution of the various As_{pl_A} classes appears random and different drumlin types are found next to each other. Coordinate system is in UTM (zone 19N), Datum is WGS84.

fields, mapped in different regions worldwide, were analysed and they all show generally consistent and similar results, with only small local variations.

3. The pre-conceived idea (from the literature) of how drumlin planar shape "should look" may have affected the mapper's ability to objectively identify drumlin outlines. They may, for example, have preferentially sought out classic forms or forced other shapes into this template. However, drumlins were mapped by five people with different glaciological backgrounds and without knowing that this database would be specifically used for analysing drumlin planar shape asymmetry. The results obtained in this work show that the classic drumlin shape is, in fact, quite rare, which in itself, is an obvious indication that such an influence did not generally occur.
4. *A priori* knowledge of the palaeo-ice flow direction of the mapped area may have biased the mapping towards one type of asymmetry against the opposite. However, the majority of drumlins were mapped (at least in Britain, which dominates the sample) without knowing the palaeo-ice flow direction of the area.
5. The presence of extremely elongated drumlins which may bias the results of As_{pl_A}. Both parameters introduced here (As_{pl}, As_{pl_A}) were developed by having in mind a "usual" drumlin elongation (EL = length/width) of approximately 3 (all figures in this paper show drumlins with that elongation), which is in fact the average value

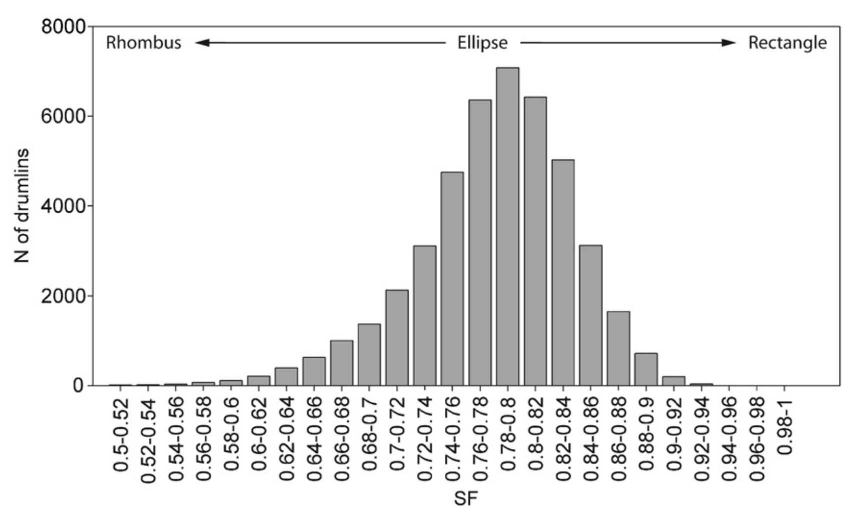

Fig. 12. Histogram (number of drumlins) of drumlin parameter *SF* in classes of 0.02 *SF*-units. The modal value (approximately 7000 drumlins) is 0.8–0.82. The overall distribution indicates a longer tail towards lower *SF* values (negative skewness).

of most mapped drumlins in Britain (Clark et al. 2009). In the rare case of drumlins that are much more elongated, the parameter As_{pl_A} could become inappropriate, tending to values close to 0.5 regardless of drumlin true asymmetry and geometry. However, drumlins with an elongation (EL) high enough to influence this parameter are actually very rare in the database (e.g. only 10% of drumlins have an EL>5). Moreover, when the same two parameters (As_{pl}, As_{pl_A}) were quantified only for drumlins with an EL<3, the average results were nearly identical to those obtained by analysing the entire database without any filter.

6. Conclusion

The longitudinal asymmetry of planar shape has been a widely accepted property of drumlins that has inspired theories of drumlin formation. Given its widespread acceptance, it is remarkable to note the almost complete absence of any quantitative tests of this property in the abundant literature on drumlin morphometry. In this paper, an objective quantitative analysis of drumlin planar shape was applied to the largest drumlin database ever created; 44,500 drumlins mapped in various settings in Northern America and Northern Europe.

The surprising result is that most drumlins do not show a substantial difference in their upstream and downstream planform shapes as stated or implied in most textbooks and papers. In particular, most drumlins have their transverse axis intersecting the longitudinal axis near its midpoint. Furthermore, when drumlins are split into two halves on the longitudinal axis midpoint, most drumlins have an upstream half area as large as the downstream half area. The most common drumlin planar shape is therefore longitudinally symmetric. Classic asymmetric drumlins are, indeed, present but they are much rarer than the symmetric ones. Moreover, reversed asymmetric drumlins are found to be nearly as common as classically-shaped drumlins. These conclusions are true when considering all mapped drumlins as a whole database but they also hold when observing at the regional or drumlin field scale. In fact, classic asymmetric and reversed drumlins are found within the same drumlin field next to each other, ruling out any hope for a consistent use of drumlin planar asymmetry in inferring the palaeo-ice flow direction. The results presented in this paper indicate that the existing paradigm of drumlins typically possessing a planar shape asymmetry is false, with the implication being that any drumlin formation theory based, at least in part, on this paradigm is questionable. Stoss and lee geometries cannot therefore be used to reliably indicate palaeo-ice flow direction, and theories of drumlin generation should strive to explain why "classical" or "reversed" drumlins can occur and co-exist in a same drumlin field. More importantly, theories and models must be able to explain and predict that the large majority of drumlins have a symmetric rather than an asymmetric shape. We have shown the symmetric drumlins are the norm, but still wonder if they naturally formed with a symmetric shape or whether they evolved into that planar shape through time. In other words, can we exclude that they were actually "born" asymmetric and subsequently remoulded into a symmetric shape either by deposition or erosion? Further investigations are required to answer these open, but crucial, questions. On the morphometric side there are at least two lines of investigations worth pursuing. First is the analysis of drumlin 3D shape that will make it possible to verify whether or not this property of planar shape symmetry can be extended to the entire drumlin body. Second, a sub-regional scale analysis of drumlin shape patterns might help understand the role (if any) played by local factors in the formation of drumlins in general and in the development of their final shape in particular.

Acknowledgements

This work was supported by the UK Natural Environmental Research Council NE/D011175/1 grant to CDC on "Testing the Instability Theory of Subglacial Bedform Production". We thank our colleagues Felix Ng, Richard Hindmarsh, Andrew Fowler and Heike Gramberg for fruitful discussions. We would also like to thank the reviewers, Anders Shomacker and Mike Smith, and the editor of this special issue, Jasper Knight, for their helpful corrections, comments and suggestions. This work was undertaken whilst MS was a recipient of a University of Sheffield research associate position funded by the UK NERC NE/D011175/1 grant. ALCH was funded by a British Geological Survey NERC PhD Studentship (NER/S/A/2004/12102) and is grateful to Colm Jordan of BGS for supervisory support and BGS for access to datasets on which her mapping is based.

References

Alden, W.C., 1905. Drumlins of south-eastern Wisconsin. US Geol. Surv. Bul. 273, 1–46.
Armstrong, J.E., Tipper, H.W., 1948. Glaciation in north central British Columbia. Am. J. Sci. 246, 283–310.
Barnett, H.F., Finke, P.G., 1971. Morphometry of landforms: drumlins. Army Natick Labs Mass Earth Sci. Lab Tech. Rep. ES-63, 1–34.
Benn, D.I., Evans, D.J.A., 1998. Glaciers and Glaciation. Arnold, London. p. 431.
Boulton, G.S., 1987. A theory of drumlin formation by subglacial sediment deformation. In: Menzies, J., Rose, J. (Eds.), Drumlin Symposium. Balkema, Rotterdam, pp. 25–80.
Chamberlin, T.C., 1883. Terminal moraines of the second glacial epoch. United States Geological Survey, 3 rd annual report, pp. 291–402.
Charlesworth, J.K., 1957. The Quaternary Era. Arnold, London, pp. 389–403.
Chorley, R.J., 1959. The shape of drumlins. J. Glaciol. 3, 339–344.
Chorley, R.J., Malm, D.E.G., Pogorzelski, H.A., 1957. A new standard for estimating drainage basin shape. Am. J. Sci. 255, 138–141.
Clark, C.D., 1993. Mega-scale glacial lineations and cross-cutting ice-flow landforms. Earth Surf. Proc. Land. 18, 1–29.
Clark, C.D., Meehan, R.T., 2001. Subglacial bedform geomorphology of the Irish Ice Sheet reveals major configuration changes during growth and decay. J. Quatern. Sci. 16, 483–496.
Clark, C.D., Hughes, A.L.C., Greenwood, S.L., Spagnolo, M., Ng, F., 2009. Size and shape characteristics of drumlins, derived from a large sample, and associated scaling laws. Quatern. Sci. Rev. 28, 677–692.
Dardis, G.F., Hanvey, P.M., 1994. Sedimentation in a drumlin lee-side subglacial wave cavity, northwest Ireland. Sed. Geol. 91, 97–114.
Dyke, A.S., Prest, V.K., 1987. Late Wisconsinan and Holocene history of the Laurentide Ice Sheet. Géog. Phys. Quatern. 41 (2), 237–263.
Doornkamp, J.C., King, C.A.M., 1971. Numerical Analysis in Geomorphology: An Introduction. Arnold, London, pp. 294–304.
Ebers, E., 1926. Die bisherige Ergebnisse der Drumlinforschung. Eine Monogr. Drumlins. Neues Jahrbuch Geol. Paläontol. Abh. 53B, 153–270.
Ebers, E., 1937. Zur Entstehung der Drumlins als Stromlinienkörper. Zehn weitere Jahre Drumlinforschung (1926–1936). Neues Jahrbuch Mineralog., Geol. Paläontol. 78 B, 200–240.
Fairchild, H.L., 1929. New York drumlins. Rochester Acad. Sci. 7, 1–37.
Flint, R.F., 1957. Glacial and Pleistocene Geology. John Wiley and Sons Inc., New York. 553 pp.
Flint, R.F., 1971. Glacial and Quaternary Geology, 5. John Wiley and Sons Inc, New York, London, pp. 100–104.
Fowler, A.C., 2000. An instability mechanism for drumlin formation. In: Maltman, A.J., Hubbard, B., Hambrey, M.J. (Eds.), Deformation of Glacial Materials: Geological Society of London, Special Publications, 176, pp. 307–319.
Francek, M., 1991. A spatial perspective on the New York drumlin field. Phys. Geogr. 12, 1–18.
Gluckert, G., 1973. Two large drumlin fields in Central Finland. Fennia 120, 5–37.
Gravenor, C.P., 1974. The Yarmouth drumlin field, Nova Scotia, Canada. J. Glaciol. 13 (67), 45–54.
Greenwood, S.L., Clark, C.D., 2010. The sensitivity of subglacial bedform size and distribution to substrate lithological control. Sed. Geol. 232, 130–144 (this issue).
Harry, D.G., Trenhaile, A.S., 1987. The morphology of the Arran drumlin field, southern Ontario, Canada. In: Menzies, J., Rose, J. (Eds.), Drumlin Symposium. Balkema, Rotterdam, pp. 161–173.
Hart, J., 1999. Identifying fast ice flow from landform assemblages in the geological record: a discussion. Ann. Glaciol. 28, 59–66.
Hattestrand, C., Goodwillie, D., Kleman, J., 1999. Size distribution of two cross-cutting drumlin systems in northern Sweden: a measure of selective erosion and formation time length. Ann. Glaciol. 28, 146–152.
Hess, D.P., Briner, J.P., 2009. Geospatial analysis of controls on subglacial bedform morphometry in the New York drumlin field — implications for Laurentide ice sheet dynamics. Earth Surf. Proc. Land. 34, 1126–1135.
Hindmarsh, R.C.A., 1999. Coupled ice-till dynamics and the seeding of drumlins and bedrock forms. J. Glaciol. 28, 221–230.
Hitchcock, C.H., 1876. Lenticular hills of glacial drift. Proc. Boston Soc. Nat. Hist. 19, 63–67.
Kleman, J., Borgstrom, I., Stroeven, A., 1997. Fennoscandian palaeoglaciology reconstructed using a glacial geological inversion model. J. Glaciol. 43, 283–299.
Kupsch, W.O., 1955. Drumlins with jointed boulders near Dollard, Saskatchewan. Geol. Soc. Am. Bull. 66, 327–338.
Menzies, J., 1979a. A review of the literature on the formation and location of drumlins. Earth Sci. Rev. 14, 315–359.

Menzies, J., 1979b. Mechanics of drumlin formation. J. Glaciol. 27, 372–384.
Mitchell, W.A., 1994. Drumlins in ice sheet reconstructions, with reference to the western Pennines, northern England. Sed. Geol. 91, 313–332.
Patterson, C.J., Hooke, L.H., 1995. Physical environment of drumlin formation. J. Glaciol. 41 (137), 30–38.
Piotrowski, J.A., 1987. Genesis of the Woodstock drumlin field, southern Ontario, Canada. Boreas 16, 249–265.
Rattas, M., Piotrowski, J.A., 2003. Influence of bedrock permeability and till grain size on the formation of the Saadjärve drumlin field, Estonia, under an east-Baltic Weichselian ice stream. Boreas 32, 167–177.
Reed, B., Galvin, C.J., Miller, J.P., 1962. Some aspects of drumlin geometry. Am. J. Sci. 260, 200–210.
Rouk, A.M., Raukas, A., 1989. Drumlins of Estonia. Sed. Geol. 62, 371–384.
Schomacker, A., Krüger, J., Kjaer, K.H., 2006. Ice-cored drumlins at the surge-type glacier Brúarjökull, Iceland: a transitional-state landform. J. Quatern. Sci. 21, 85–93.
Shaw, J., 1983. Drumlins formation related to inverted meltwater erosional marks. J. Glaciol. 29, 461–479.
Shaw, J., 1989. Drumlins, subglacial meltwater floods and ocean responses. Geology 17, 853–856.
Shaw, J., 2002. The meltwater hypothesis for subglacial bedforms. Quatern. Int. 90, 5–22.
Shaw, J., Freschauf, R.C., 1973. A kinematic discussion of the formation of glacial flutings. Can. Geogr. 17, 19–35.
Shaw, J., Kvill, D., 1984. A glaciofluvial origin for drumlins of the Livingstone Lake Area, Saskatchewan. Can. J. Earth Sci. 21, 1442–1459.
Shaw, J., Kvill, D., Rains, B., 1989. Drumlins and catastrophic subglacial floods. Sed. Geol. 62, 177–202.
Slater, G., 1929. The structure of the drumlins exposed on the south shore of Lake Ontario. NY State Mus. Bull. 281, 1–19.
Smalley, I.J., Unwin, D.J., 1968. The formation and shapes of drumlins and their distribution and orientation in drumlin fields. J. Glaciol. 7, 377–390.
Smalley, I., Warburton, J., 1994. The shape of drumlins, their distribution in drumlin fields, and the nature of the sub-ice shaping forces. Sed. Geol. 91, 241–252.
Smith, M.J., Clark, C.D., 2005. Methods for the visualisation of digital elevation models for landform mapping. Earth Surf. Proc. Land. 30, 885–900.
Smith, M.J., Clark, C.D., Wise, S.M., 2001. Mapping glacial lineaments from satellite imagery: an assessment of the problems and development of best procedure. Slovak Geol. Mag. 7, 263–274.
Stokes, C.R., Clark, C.D., 2002. Are long bedforms indicative of fast ice flow? Boreas 31, 239–249.
Stokes, C.R., Clark, C.D., 2003. The Dubawnt Lake palaeo-ice stream: evidence for dynamic ice sheet behaviour on the Canadian Shield and insights regarding the controls on ice-stream location and vigour. Boreas 32 (1), 263–279.
Trenhaile, A.S., 1975. The morphology of a drumlin field. Ann. Assoc. Am. Geogr. 65, 297–312.
Wright, H.E., 1962. Role of the Wadena lobe in the Wisconsin glaciation of Minnesota. Bull. Geol. Soc. Am. 73, 73–100.
Wysota, W., 1994. Morphology, internal composition and origin of drumlins in the southeastern part of the Chelmno–Dobrzy Lakeland, North Poland. Sed. Geol. 91, 345–364.
Zakrzewska Borowiecka, B., Erickson, R.H., 1985. Wisconsin drumlin field and its origin. Z. Geomorphol. 29 (4), 417–438.